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Building modeling as a crucial part for building predictive control☆

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ABSTRACT

Recent results show that a predictive building automation can be used to operate buildings in an energy and cost effective manner with only a small retrofitting requirements. In this approach, the dynamic models are of crucial importance. As industrial experience has shown, modeling is the most time-demanding and costly part of the automation process. Many papers devoted to this topic actually deal with modeling of building subsystems. Although some papers identify a building as a complex system, the provided models are usually simple two-zones models, or extremely detailed models resulting from the use of building simulation software packages. These are, however, not suitable for predictive control. The objective of this paper is to share the years-long experience of the authors in building modeling intended for predictive control of the building's climate. We provide an overview of identification methods for buildings and analyze their applicability for subsequent predictive control. Moreover, we propose a new methodology to obtain a model suitable for the use in a predictive control framework combining the building energy performance simulation tools and statistical identification. The procedure is based on the so-called co-simulation that has appeared recently as a feature of various building simulation software packages.

1. Introduction

1.1. Motivation for advanced control in buildings

Building climate control has drawn a lot of attention in recent years in both academia and industry. Buildings account for 20–40% of the total final energy consumption, and in the developed countries, the amount per year increases at a rate 0.5–5% [1]. In addition, the building sector is responsible for 33% of global CO_2 emissions. The savings related to buildings are therefore a natural objective of many research groups. Apart from retrofitting and modernization, one of the most popular current approaches is the application of advanced control strategies to the building automation systems (BAS) or to some of their parts.

1.2. Current control approaches, trends and possible improvements

Even though a number of advanced control solutions have been suggested by researches, the most widely used method in building temperature control has been until recently a controller supervised by heating-curve (HC) which require no model of the process (see e.g. [2,3]). The respective subsystems of heating, ventilation, and air conditioning (HVAC) are then controlled making use of rule-based controllers (RBC, "if–then–else") [4], which are mainly responsible for a specific and space-limited area. On the level of the whole building, there is no optimization (even though there are often highly sophisticated local controllers). This is caused by extreme complexity of the respective RBCs and the fact that it is practically impossible to generalize their rules for the building level. This problem becomes even more severe in view of the rising complexity of BAS tasks in modern office buildings.

One can distinguish two main research directions in advanced HVAC control (i) learning based approaches of artificial intelligence (AI) like neural networks, genetic algorithms, fuzzy techniques, support vector machines, etc. (ii) Model predictive control (MPC) techniques that stand on the principles of classical control. Generally, learning based techniques are easier to implement (if lots of on-site measurements are available) but the subsequent AI model is not suitable for optimization, lacks a physical insight and does not deal well with changes as caused by varying occupancy behavior or physical changes in the building.

MPC is a well established method for constrained control and has also been in focus of researchers in the area of buildings [5–9]. Among the first notes about MPC for supervisory control of a building was the work presented by [10], however, due to the

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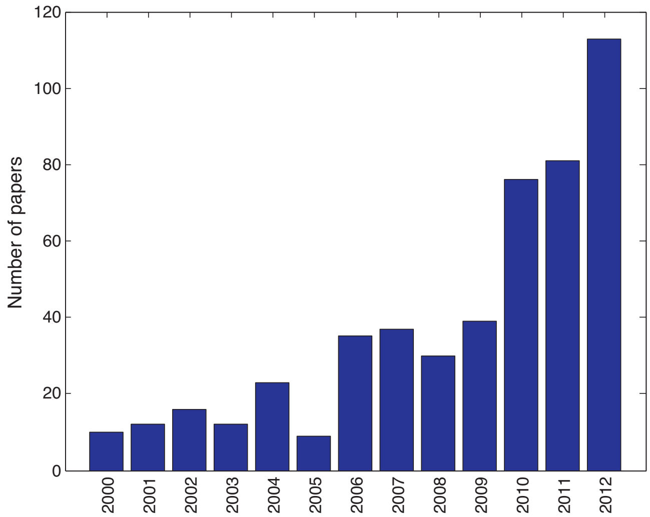

Fig. 1. Number of papers devoted to MPC in buildings in journals Energy and Buildings, Building and Environment and Energy.

computational demands, this framework has not received much attention until the past decade when MPC was applied to various types of buildings systems often using standard simulation tools. The growing interest in the use of MPC for buildings is well demonstrated by Fig. 1. Lately, the concept of predictive control has found a way to the practical applications as well [5,11,12].

MPC opens up possibilities of exploiting thermal storage capacities. It makes use of prediction of future disturbances (internal gains due to people and equipment, weather, etc.) given requirements such as comfort ranges (single value set-points still remains possible to set) for controlled variables. The control ranges (constraints) are either known in advance or at least estimated for controlled variables, disturbances, control costs, etc.

1.3. Dynamic model as a crucial part of MPC

Reliable predictions from the identified dynamic model are crucial for a sound performance of MPC. It is a well-known fact that modeling and identification are the most difficult and time-consuming parts of the automation process as such [13], particularly for predictive control. The basic conditions that each model intended for MPC usage should satisfy are reasonable simplicity, well estimated system dynamics and steady-state properties as well as satisfactory prediction properties. These requirements do not need to be of the same quality on the whole frequency range, rather they should comply with the quality requirements for the control-relevant frequency range (see e.g. [14–16]). The key question therefore is what kind of model should be searched for?

Two basic paradigms to derive a total model of building dynamics are at hand. The first one originated in HVAC engineering and building automation communities, a “traditional” approach, which uses knowledge of the structure and physical and material properties of a building. A detailed building model is then assembled from simple subsystems mutually physically interacting, making use of computer aided modeling tools, e.g. Trnsys [17], EnergyPlus [18], ESP-r [19], etc. Their objective is to simulate the behavior of the building, however, they do not provide an explicit model,[1] thus can be hardly classified into control oriented modeling approaches even though there is a challenging project GenOpt aiming at employing a (predictive) control framework directly without the need of a simple model [20]. This is however very computationally demanding, hardly scalable and therefore not further considered here.

An alternative is to use statistically based, i.e. data-driven approaches, resulting in a model in an explicit form. We must emphasize that even physically-based parametric models are classified into statistically-based models here as the parameters are identified using measured or simulated data.

Basically, following categories of building modeling techniques suitable for predictive control that can be considered as statistical.

Subspacemethods(4SID) [21] belong to the black-box identification algorithms and provide a model in a state space form.

The main advantage of 4SID methods is their ability to handle large amount of data. This was demonstrated for instance in the identification of a thermodynamic model of a small residential building that was equipped with tens of wireless sensors collecting temperatures, humidity and solar radiation [22]. 4SID methods were also used for an identification of a university building: at first, the authors compared prediction error methods with 4SID methods [23], then showed that a suitable identification experiment can significantly increase quality of the resulting model [24] as the quality of input–output data is a key factor for 4SID methods. Further on, 4SID algorithm was also applied for the identification of a large office building [25].

Predictionerrormethods(PEM) [26] are the most commonly used statistical identification techniques. Their objective is to minimize one-step ahead prediction error by optimizing parameters of a prespecified model structure.

Typically, autoregressive moving average with external input (ARMAX) model structures are preferred. This structure is used for modeling of a room temperature in office buildings as presented in [27], the model is then used for real-time fault detection and control applications. In [28], several black-box model structures are investigated for identification of the thermal behavior of a modern office building. The authors conclude that Box–Jenkins general model results in the best prediction performance among the studied group.

PEM are simple-to-use methods that are, however, suitable mainly for identification of single-input single-output (SISO) systems. As the building systems are normally multiple-input multiple-output (MIMO) systems, these methods have to be carefully used. In [29], the authors show that modeling of air conditioning process by multiple SISO ARMAX models of all system components leads to poor performance compared to the proposed MIMO ARMAX counterpart.

MPCrelevantidentification(MRI) is an approach minimizing multi-step ahead prediction errors [30–32]. The horizon for error minimization commensurate with the prediction horizon of the predictive controller.

A multi-step ahead prediction error cost function for selection of a building model is examined in [33]. The authors adapts the MRI algorithm for usage on building data that are usually highly correlated and then show that the proposed algorithm results outperforms standard one-step ahead PEM methods.

Deterministicsemi − physicalmodeling(DSPM) uses resistance capacitance (RC) network analogue to an electric circuitry to describe the process dynamics and is often referred to as a gray-box modeling.

This approach was presented in a wide variety of papers. Gray-box technique is used to obtain a model of a university building in [11]. With this model, the MPC applied in a real operation saved 16–28% energy compared to the previous well-tuned conventional control strategy. RC networks are also used by the leading projects dealing with predictive control of buildings, i.e. UC Berkeley [5], ETH Zurich [34], KU Leuven [6].

[1] Note that in this context, we call a model explicit if there are mathematical formulas describing a state evolution, i.e. a set of differential or difference equations is available. Otherwise the model is called implicit. Notice that AI models are also implicit.

Besides that, [35] shows how to use identification toolbox for Matlab [36] to estimate parameters of RC networks, while in [37], the authors estimate building parameters using genetic algorithms minimizing one-step prediction error. Detailed RC network for thermally activated building systems (TABS) is presented in [38].

Probabilisticsemi – physicalmodeling(PSPM) [39] utilizes stochastic differential equations for the description of a system to be identified. A hierarchy of models with increasing complexity is formulated based on prior physical knowledge, parameters of each model are estimated using the maximum likelihood (ML) method and a forward selection strategy is used to find a meaningful and adequately complex model by an iterative process.

This technique is presented in a series of papers by [40,41], the additional statistical tests for the iterative procedure are proposed in [42].

1.4. The contribution and a structure of the paper

Following the discussion in the previous paragraphs, MPC has a potential to address the issue of energy consumption in buildings as well as growing complexity of control requirements. The crucial part of MPC is the dynamic model. The objective of this paper is to (i) present a review of methods applicable to the building modeling intended for the predictive control and (ii) address an issue of handling of the growing complexity of modern buildings.

The paper continues with the following structure. The next section is devoted to building modeling and identification approaches – those well-known in control engineering community as well as those originating from the community of building and civil engineers. Section 3 is devoted to a novel method combining a building modeling software with a subsequent statistical identification. This approach can be conveniently used for large office buildings. Section 4 presents two case studies: the first is an artificial example of a simple, yet realistic building constructed in Trnsys environment, where the properties of several identification approaches are shown, while the second is a statistically-based identification of a large office building in Munich. To the best of the authors' knowledge, there was no detailed building modeling of such size intended for predictive control as is discussed there. The last section contains final remarks and concludes the paper.

1.5. Notation

Throughout the paper $\mathbb{R}$ denotes the set of reals, $\mathbb{Z}$ set of integers, $t \in \mathbb{R}$ the time while $k \in \mathbb{Z}$ is the discrete time, vectors $u \in \mathbb{R}^m$, $x \in \mathbb{R}^n$, $y \in \mathbb{R}^r$ stand for system input, state and output, respectively. The symbols $w \in \mathbb{R}^n$ and $e \in \mathbb{R}^r$ denote process and measurement noise sequences, respectively. The positive integer N stands for number of identification data while P is the length of an MPC prediction horizon. Notation Z_1^j means that matrix Z_1^j is composed as $Z_1^j = [z_1, z_2, \ldots, z_j]$. The symbol $(\cdot)^\dagger$ stands for Moore–Penrose pseudo-inverse of a matrix, whilst the symbol $\hat{M}$ means the estimate of quantity M. The symbol I_s stands for the identity matrix of size s. Finally, the symbols vec$(\bullet)$ and $\otimes$ denote the vectorization and the Kronecker product, respectively.

2. Modeling and identification for buildings

In the following, two basic concepts for derivation of a building model are treated in detail. First, we deal with an approach using a building simulation software, and thereafter we will have a look at statistically-based approaches.

2.1. Physically-based models, simulation tools

Physically-based models are typically developed making use of specialized computer aided modeling tools. The particular tool then assembles the model from the provided information about building structure and physical and material properties. A detailed building model is then assembled from simple subsystems mutually physically interacting. Overall, the software packages are called building energy performance simulation tools (BEPST).

2.1.1. Application of a building energy performance simulation

The building energy performance simulation has become an important tool to assess the building's energy consumption and user comfort. In early design phases architects and designers mostly use BEPST to compare performance of different design variants. The simulation inputs are based on the designer's experience since not all design decisions are finalized yet. The building's energy performance simulated at these stages may vary greatly from the actual building's energy performance once it is in operation. It is however possible to deduce tendencies of expected performance of different design solutions in early design stages.

Application of BEPST is not limited to the early design phases. In more detailed design phases the building simulation is often used to check the functionality of a proposed design and increase the planning reliability. In addition, the building simulation is increasingly employed to evaluate an absolute energy performance which requires a greater level of detail for all energy consuming building and plant components.

The level of detail for modeling HVAC plants depends on the available building data. For gathering all the information needed to model a building and its plants, a close cooperation of technical consultants for architecture, HVAC and electrical and the building owner is necessary. In early design phases, issues of building control are often postponed to detailed planning. Simulation engineers thus often fall back on implementing standardized and simplified control rules in their models.

2.1.2. Control in building energy performance simulation tools

Currently available BEPST have quite different control capabilities. Typically, simulation tools provide thermostat and humidistat control as well as pre-defined control strategies for system availability and plant control. Because BEPTS use idealized approximations, control in simulation tools performs more efficient and stable as it might be in real-world applications. Thus, the calculated building energy consumption is generally optimistic [43].

2.2. Statistically-based identification approaches

The building modeling approaches described in the following are in-line with the short discussion from Section 1.3.

2.2.1. Subspace identification

Buildings usually have tens or even hundreds of rooms/zones with a large number of actuators and sensors, what results in MIMO model.

One of the most popular and successful methods for identification of MIMO systems is subspace state-space system identification (4SID).

Problemstatement. The objective of the 4SID is to find matrices A, B, C, D and K of a linear time-invariant (LTI) discrete-time model in an innovative form:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ke_k, \\ y_k &= Cx_k + Du_k + e_k \end{aligned} \tag{1}$$

Table 1
Symbols and their meaning used for 4SID algorithm.

Symbol	Meaning
Y_p	Hankel matrix of the past outputs
Y_f	Hankel matrix of the future outputs
X_p	Hankel matrix of the past states
X_f	Hankel matrix of the future states
U_p	Hankel matrix of the past inputs
U_f	Hankel matrix of the future inputs
Γ_i	Extended observability matrix
H^d	Markov parameter matrix corresponding to the deterministic part
H^s	Markov parameter matrix corresponding to the stochastic part
Δ^d	Reversed extended controllability matrix corresponding to the deterministic part
Δ^s	Reversed extended controllability matrix corresponding to the stochastic part
E_p	Hankel matrix of the past noise
E_f	Hankel matrix of the future noise
i	Number of block rows in Hankel matrices

based on given measurements of an input and an output generated by an unknown stochastic system of order n subject to unknown white noise e_k.

Algorithm. The entry point is the input–output equations:

$$\begin{aligned} Y_p &= \Gamma_i X_p + H_i^d U_p + H_i^s E_p, \\ Y_f &= \Gamma_i X_f + H_i^d U_f + H_i^s E_f, \end{aligned} \tag{2}$$

$$X_f = A^i X_p + \Delta_i^d U_p + \Delta_i^s E_p, \tag{2}$$

where the symbols are defined in Table 1. Note that a very detailed explanation of the respective symbols, e.g. how the Hankel matrices are constructed is provided in [21,44]. The basic idea of the algorithm is to drop input and noise matrices by finding an appropriate projection and instrument matrices. The main tool of 4SID is an oblique projection defined as follows [21]:

$$\mathfrak{Z} = Y_f \underset{U_f}{/} W_p = Y_f \begin{bmatrix} W_p^T & U_f^T \end{bmatrix} \begin{bmatrix} W_p W_p^T & W_p U_f^T \\ U_f W_p^T & U_f U_f^T \end{bmatrix}^{\dagger} \begin{bmatrix} I_r \\ 0 \end{bmatrix} W_p, \tag{3}$$

where $W_p = (U_p/Y_p)$, i.e. the matrices of past inputs and outputs are stacked onto each other. The equation basically represents the projection of future system outputs onto a space of past system inputs. Then it can be shown [21] that $\mathfrak{Z} = \Gamma X$, where X is the Kalman filter state sequence and Γ is state observability matrix. The order of the system can be determined from an analysis of singular values obtained from a singular value decomposition (SVD) of $W_1 \mathfrak{Z} W_2$, where $W_{1,2}$ are weighting matrices of an appropriate size which determine the resulting state space basis as well as the importance of the particular element of $\mathfrak{Z}$, see Eq. (8).

The algorithm continues from either Γ or X in a slightly different manner depending on the particular subspace identification algorithm, however, both ways lead to a computation of A and C by ordinary least squares (OLS).

For selection of a submatrix we have adopted a Matlab-like notation, where $A(1:n, :)$ means, that the submatrix is obtained from the original matrix A by taking 1 to n rows and all the columns. Then

$$\hat{C} = \Gamma(1:r, :), \tag{4a}$$

$$\hat{A} = \Gamma(1:(i-1)*r, :)^{-1}\Gamma(r+1:i*r, :). \tag{4b}$$

Given $\hat{A}$ and $\hat{C}$, the estimate of B and D (and an initial state x_0) is performed in different ways [21,26,45–47]; here the general idea will be outlined. The system output equation of Eq. (1) can be written as:

$$y_k = CA^k x_0 + \sum_{j=0}^{k-1} CA^{k-j-1} Bu_j + Du_k + e_k. \tag{5}$$

Note that in this equation, the only unknowns are x_0 and matrices B and D. The rest of the terms are known or can be replaced by the estimates. With aid of vectorization and Kronecker product, the equation can be rewritten into a form of a least-squares problem. For more details, consult [48].

Finally, given the estimates of A, B, C, D, the Kalman gain matrix K can be computed solving the Algebraic Riccati Equation (ARE) in which the covariance matrices Q, S and R:

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} = \frac{1}{N} \left(\begin{bmatrix} W \\ V \end{bmatrix} \begin{bmatrix} W^T & V^T \end{bmatrix} \right) \tag{6}$$

are determined from the residuals as follows:

$$\begin{bmatrix} W \\ V \end{bmatrix} = \begin{bmatrix} X_{k+1} \\ Y_k \end{bmatrix} - \begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix} \begin{bmatrix} X_k \\ U_k \end{bmatrix} \tag{7}$$

At last, a short note on choice of a system order is given. Two possible approaches are at hand.

- The oblique projection matrix is decomposed by SVD and then the system order is determined as a number of non-zero singular values of Σ matrix in $SVD(W_1 \mathfrak{Z} W_2) = \mathcal{U}\Sigma\mathcal{V}^T$,
- In case of worse signal to noise ratio, the estimation of the number of dominant singular values becomes cumbersome. An alternative heuristic approach improving the order estimation is suggested as:

$$\begin{aligned} f(\sigma_j) &= \text{grad}\log(\sigma_j) \quad j = 1, \ldots, i \cdot r, \\ n &= \underset{j}{argmin} f(\sigma_j), \end{aligned} \tag{8}$$

where σ_j are singular values of Σ. Note that this heuristic can be used in situations, when there is a low signal to noise ration, thus the singular values are "drowned" in the noise.

Subspaceidentificationforbi − linearsystems. Some phenomena in buildings cannot be modeled using linear physics by their nature. These are, for instance, operation of ventilation units [49] or the heat transmission through the windows [34]. The latter is caused by opening and closing the blinds (which can be controlled by MPC). The effects of the blinds on the dynamics can be modeled by splitting the heat transmission [34]. The first part describes the heat transmission with closed blinds (constant), whilst the second part describes the heat transmission with the partially or fully opened blinds. This means, that for the partially or fully opened blinds, the system state is multiplied by an input $u \in \{0, 1\}$, which forms a bi-linear system description. A product of mass flow rate and a temperature (to obtain a heat flux) results in another example of a bi-linearity. Bi-linearities are treated in detail in [5].

A possible solution is to use bi-linear subspace algorithm (Bi4SID). The objective of Bi4SID is to find a bi-linear, time-invariant, discrete time model in a form:

$$\begin{aligned} x_{k+1} &= Ax_k + F(x_k \otimes u_k) + Bu_k + w_k, \\ y_k &= Cx_k + Du_k + e_k. \end{aligned} \tag{9}$$

The objective of the algorithm is to determine the system order n and to find the matrices A, B, D, C and F up to some similarity transformation.

The biggest disadvantage of the Bi4SID is that the number of rows of data matrices grows exponentially with the order of the system [50,51]. This drawback was to a great extent overcome by kernel method [51,52], where used kernel data matrices have smaller dimensions than those used in original bi-linear subspace problem. As an alternative to the kernel method, the basic OLS problem has been reformulated as a Ridge regression problem [52], where for solution the only kernel matrix is needed. Even with these simplifications, Bi4SID are not yet applicable to the real building as they are unable to process larger amounts of data.

2.2.2. Prediction error methods

Prediction error methods [26] (PEM) are frequently used for system identification and can be formulated as:

$$\hat{\theta} = \operatorname{argmin}_{\theta} \sum_{k=1}^{N} \ell(\varepsilon_k(\theta)), \tag{10}$$

where $\ell(\bullet)$ is an appropriate scalar-valued function, θ is a vector of parameters and ε_k a prediction error in time k, $\varepsilon_k = y_k - \hat{y}_k$, with $\hat{y}_k$ denoting the output estimate. Typically, one-step ahead prediction is to be minimized which uses past output data up to time $k-1$ to obtain the estimate of y_k. This is formally written as $\hat{y}_{k|k-1} = f(U_1^k, Y_1^{k-1})$. The function f depends on the user's choice of model structure (ARX, ARMAX, etc.).

2.2.3. Model predictive control relevant identification

When building-up a model for MPC, we should think about the minimization of the control error on the prediction horizon. Hence a model used for predictive control should be primarily a sound multi-step predictor. Such methods, minimizing the multi-step prediction error, are collectively called MPC relevant identification methods (MRI) [32,53–55] and in some sense extend PEM. These methods are addressed in detail in the following.

Problemstatement. A possible formulation of a basic MPC problem can be as follows:

$$\min_{u_0,\ldots,u_{P-1}} \sum_{k=0}^{P-1} (y_k^{ref} - y_k)^T Q_k (y_k^{ref} - y_k) + R_k u_k, \tag{11}$$

$$\text{subject to :} \quad x_0 = x, \tag{12}$$

$$x_{k+1} = f(x_k, u_k), \tag{13}$$

$$y_k = g(x_k, u_k), \tag{14}$$

$$(x_k, u_k, y_k) \in \mathcal{X}_k \times \mathcal{U}_k \times \mathcal{Y}_k, \tag{15}$$

where $\mathcal{X}_k, \mathcal{U}_k$ and $\mathcal{Y}_k$ denote the constraints sets of states, inputs and outputs. Q_k and R_k are time varying weighting matrices of appropriate dimensions. Based on Eq. (11), without penalization on control, the MPC cost function which penalizes the sum of the squared differences of the actual value of the controlled output y_k and the required reference y_k^{ref} during a prediction horizon can be rewritten as:

$$J_{MPC} = \frac{1}{(N-P)P} \sum_{k=1}^{N-P} \sum_{i=1}^{P} (y_{k+i}^{ref} - y_{k+i})^2 \tag{16}$$

For buildings, P is typically chosen such that it corresponds to 48 h, while N is significantly larger. Next, $y_{k+i} = \hat{y}_{k+i|k} + e_{k+i|k}$, where $\hat{y}_{k+i|k}$ denotes the predicted output values at the time $k+i$ using the data until k, $e_{k+i|k}$ is the i-step ahead prediction error. Eq. (16) can be rewritten [53] as:

$$\begin{aligned} J_{MPC} = &\frac{1}{(N-P)P} \sum_{k=1}^{N-P} \sum_{i=1}^{P} (y_{k+i}^{ref} - y_{k+i|k})^2 \\ &+ \frac{1}{(N-P)P} \sum_{k=1}^{N-P} \sum_{i=1}^{P} (y_{k+i} - \hat{y}_{k+i|k})^2 \\ &- \frac{2}{(N-P)P} \sum_{k=1}^{N-P} \sum_{i=1}^{P} (y_{k+i}^{ref} - \hat{y}_{k+i|k})(y_{k+i} - \hat{y}_{k+i|k}). \end{aligned} \tag{17}$$

The MPC itself minimizes only the first term. However, from global perspective, to achieve the optimal solution it is necessary minimize the remaining terms as well. The last term represents the cross-correlation between the identification and control errors and is treated by [56]. The second term in Eq. (17) will be used as an identification loss function for MRI and expresses the identification error:

$$J_{MRI} = \frac{1}{(N-P)P} \sum_{k=1}^{N-P} \sum_{i=1}^{P} \|e_{k+i|k}\|^2 = \|E_a\|^2, \tag{18}$$

or with explicit dependence on estimated parameters as:

$$J_{MRI}(\Theta) = \|E_a\|^2 = \|Y_a - Z_a(\Theta)\Theta\|^2 \tag{19}$$

with

$$E_a = \begin{bmatrix} E_{a_1} \\ \vdots \\ E_{a_P} \end{bmatrix}, \quad E_{a_i} = \begin{bmatrix} e_{1+i|1} \\ \vdots \\ e_{N|N-i} \end{bmatrix}, \quad i = 1, \ldots, P \tag{20}$$

and similarly defined output matrix Y and regressor Z. The specific form of regressor depends on the model used.

2.2.4. Estimation of ARX models

In case that AutoRegressive eXternal input (ARX) [26] model is considered, the multi-step output prediction $\hat{y}_{k+i|k}$ is expressed as:

$$\hat{y}_{k+i|k} = Z_{k+i}\hat{\Theta}, \quad i = 1, 2, \ldots, P. \tag{21}$$

where $Z_{k+i} = [u_{k+i-n_k}, \ldots, u_{k+i-n_b}, y_{k+i-1}, \ldots, y_{k+i-n_a}]$ and $\hat{\Theta} = [\hat{b}_{n_k}, \ldots, \hat{b}_{n_b}, \hat{a}_1, \ldots, \hat{a}_{n_a}]^T$, n_b and n_a are the numbers of lagged inputs and outputs, n_k represents the relative lag of outputs w.r.t. to inputs. As the outputs y_{k_0} in Z_{k+i} with $k_0 > k$ are not available at k, the output prediction $\hat{y}_{k_0|k}$ is obtained recursively from Eq. (21), i.e. by an iterative use of one-step ahead predictions. Having formed the Z_a and Y_a according to Eq. (20), the problem can be solved by available solvers minimizing Eq. (19).

2.2.5. Estimation of state space models

When minimizing Eq. (19) for MIMO system, the use of the state space representation is more convenient than e.g. ARX parametrization. In the simplest case when all the states are measurable, the relation between $\hat{\Theta}$ and system matrices A and B can be expressed as:

$$\Theta = \begin{bmatrix} A \\ B \end{bmatrix}, \tag{22}$$

that is, if all the states are measured (C is a unit matrix), matrices A and B can be readily extracted from $\hat{\Theta}$.

The more difficult situation is for the case when some states are not measured and the particular input and output pair is represented by a higher-order transfer function $n_b > 1$ for the jth input.

The basic idea is to introduce artificial outputs (by means of A_{aux} and B_{aux}) and make thus all the states "measurable". The respective parameters are estimated by MRI minimizing Eq. (19) using MIMO ARX structure.

Without loss of generality, let us assume that the output which depends on the lagged input is the first one. Then $n_b - 1$ auxiliary variables in matrices A_{aux} and B_{aux} are introduced:

$$\begin{bmatrix} x_{n_o+1,k+1} \\ \vdots \\ x_{n_o+n_b-1,k+1} \end{bmatrix} = A_{aux} \begin{bmatrix} x_{n_o+1,k} \\ \vdots \\ x_{n_o+n_b-1,k} \end{bmatrix} + B_{aux}u, \tag{23}$$

where A_{aux} and B_{aux} are in the following form:

$$A_{aux} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \vdots \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & & \cdots & 0 \end{bmatrix}, \tag{24}$$

$$B_{aux} = \begin{bmatrix} \mathbf{0}_{j-1} & \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} & \mathbf{0}_{n_i-j} \end{bmatrix}, \tag{25}$$

with $\mathbf{0}_{j-1}$ and $\mathbf{0}_{n_i-j}$ being appropriate size zero matrices and n_i and n_o being number of inputs and outputs, respectively. Then, the system matrices $\bar{A}, \bar{B}$ can be expressed as:

$$\bar{A} = \begin{bmatrix} A & \begin{bmatrix} b_{n_b,j} & b_{n_b-1,j} & \cdots & b_{2,j} \\ 0 & 0 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & & 0 \end{bmatrix} \\ \mathbf{0} & A_{aux} \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ B_{aux} \end{bmatrix} \tag{26}$$

with A and B in Eq. (26) computed analogously to Eq. (22) leaving out the coefficients corresponding to the influence of the lagged input (this is equivalent to $n_a = n_b = 1$). The skipped coefficients are stored in the last $n_b - 1$ elements of the first row of $\bar{A}$. Note that j denotes the lagged input channel. Similar procedure is used in the case when more than one output is affected by the lagged input. Matrix C is a matrix with as many rows as system outputs (original, without artificial outputs created by introducing the auxiliary states) and as many columns as system states. Matrix D is a zero matrix.

2.2.6. Deterministic semi-physical modeling

As already stated, DSPM uses RC network analogue to electric circuitry to describe internal workings. To detail this approach, we first need to consider all kinds of heat transfers to assemble a detailed first principles model.

Conduction is a heat transfer through walls (solid body) expressed as [57]:

$$\dot{T}_2 \approx \frac{T_1 - T_2}{k_{cd}} \approx \frac{\dot{Q}}{k_{cd}}, \tag{27}$$

where T_1 is a temperature of a source, T_2 is a measured temperature of some entity, $\dot{Q}$ is a heat flux and k_{cd} stands for the conduction time constant of a process ($R \times C$ with R and C being the thermal resistance and capacity of a mass).

Convection is a heat transfer through air (liquid) expressed as:

$$\dot{T}_2 \approx \frac{T_1 - T_2}{k_{cv}} \cdot \sqrt[4]{\frac{T_1 - T_2}{T_1 + T_2}} \tag{28}$$

with a time constant k_{cv}. Eq. (28) can also be approximated by $\dot{T}_2 \approx (T_1 - T_2)/K_{cv}$ as $\sqrt[4]{(T_1 - T_2)/(T_1 + T_2)}$ is considered constant for a building heating process [57].

Radiation corresponds (similar as a convection) to a heat transfer through air and is expressed as:

$$\dot{T}_2 \approx \frac{T_1^4 - T_2^4}{k_{ra}} \tag{29}$$

with time constant k_{ra}.

Based on the simplified equations for all heat transfers, differential equations can be formulated for all states/nodes. This is schematically outlined in Fig. 2. Control actions are introduced in two ways. The first one involves simply adding a heating or cooling input to the particular room node, which then appears in the right-hand side of above mentioned equations. The second way of introducing control actions is by assuming that some resistances are variable. For example, solar heat gains and luminous fluxes through windows are assumed to vary in a linear fashion with a blind position, i.e. the corresponding resistance was multiplied with an input $u = \{0, 1\}$. This leads to a bi-linear model, i.e. bi-linear in a state and input and a disturbance and input as well.

Now we will briefly outline a simple procedure how to estimate parameters of a RC network. Other procedures exist and are usually based on the computationally demanding parameter estimation of differential equations that are solved by sequential quadratic programming. We rather present numerically simple and stable procedure based on least squares technique. The procedure was firstly used by [11].

DSPMestimationprocedure. Having described a physics of a building by a set of differential equations, the estimation problem is formulated in the continuous time. Most of the mathematical tools, however, work with discrete-time counterparts, therefore the original continuous-time problem must be reformulated to a discrete world, e.g. as:

$$\begin{aligned} A &= e^{A_c T_s} = I_n + A_c T_s + \frac{A_c^2 T_s^2}{2} + \ldots \approx I_n + A_c T_s, \\ B &= \int_0^{T_s} e^{A_c \tau} d\tau \approx \int_0^{T_s} I_n d\tau B_c = T_s B_c, \end{aligned}$$

where A_c, B_c and A, B are model matrices of continuous- and discrete-time models, respectively. T_s stands for a sampling time. This corresponds to the Euler's discretization, thus can be applied for non-linear systems as well. Then the state equation $x_{k+1} = Ax_k + Bu_k + e_k$ developed over the time can be written as:

$$X_2^N = AX_1^{N-1} + BU_1^{N-1} + E_1^{N-1} = \tag{30}$$

$$= \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} X_1^{N-1} \\ U_1^{N-1} \end{bmatrix} + E_1^{N-1} \tag{30}$$

For standard optimization using OLS, Eq. (30) is rewritten as:

$$\mathrm{vec} X_2^N = \left(\begin{bmatrix} X_1^{N-1} \\ U_1^{N-1} \end{bmatrix} \otimes I_n \right)^T \mathrm{vec} \begin{bmatrix} A & B \end{bmatrix} + \mathrm{vec} E_1^{N-1}.$$

Extra lines for a structure preservation of A and B as well as other required constraints can be added into the regressor matrix and the left-hand side matrix. Then, the unknown parameters are estimated using a weighted LS technique.

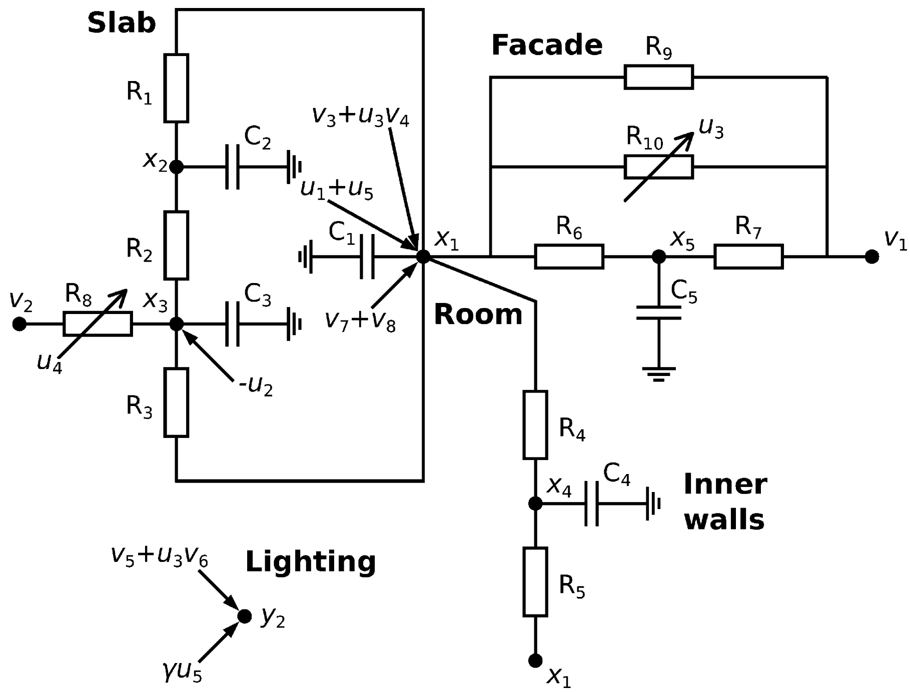

Fig. 2. RC network representing a building model.

2.2.7. *Probabilistic semi-physical modeling*

Ubiquitous noise and non-linearities in the identification data that cannot be modeled using RC networks can be partly compensated by introduction of noise additively entering system state (process noise w) and affecting measurement (measurement noise e). Hence, the RC network gets the form of a stochastic differential equation. Model parameters can then be estimated using ML technique as

$$\theta^*_{ML} = \arg\max_{\theta}\{\ln(L(\theta, Y_1^N|y_0))\}, \tag{31}$$

$$L(\theta, Y_1^N|y_0) = \prod_{k=1}^{N} \frac{\exp(-1/2\varepsilon_k^T R_{k|k-1}^{-1}\varepsilon_k)}{\left(\sqrt{2\pi}\right)^r \sqrt{\det(R_{k|k-1})}} p(y_0|\theta) \tag{31}$$

including a prior knowledge of the system. Following the standard notation, L is a likelihood function, y_0 is the vector of initial conditions, θ is the vector of unknown parameters, $p(y_0|\theta)$ is the conditional probability of initial conditions on parameters, ε_k are residuals and $R_{k|k-1}$ is a residual covariance matrix. It must be noted here, that the problem can be solved only in an iterative manner, when ε_k and $R_{k|k-1}$ are computed given an estimate $\hat{\theta}$ of θ. However, to compute $\hat{\theta}$, the knowledge of the noise properties must be assumed. The estimation of both parameters and covariance matrix is performed using the expectation maximization (EM) algorithm [58,59].

The above-mentioned procedure is iterative and interactive at the same time. Basically, at each step a model designer specify a tentative model structure $\mathcal{M}$ with several unknown parameters:

$$\overbrace{x_t = x_0 + \int_{t_0}^{t} m(\tau, x, u, p(c,\theta))d\tau + \int_{t_0}^{t} \sigma(\tau, p(c,\theta))d\beta,}^{\mathcal{M}}$$
$$y_{t_k} = h(t_k, x, u, p(c,\theta)) + R(t_k, p(c,\theta))w(t_k),$$

where β is the Wiener process, y is the vector of output measurements. $p(c, \theta)$ represents all the known and unknown parameters with c and θ being known constants and unknown parameters to be estimated, respectively; $w(t_k)$ is the Gaussian zero-mean white noise with unit variance scaled arbitrary by $R(t_k, p(c, \theta))$. Note that t_k are not necessarily uniformly spaced sampling instances. The parameter optimization then takes place and terminates when the tentative model gives a statistically relevant output response. If there are no such parameter values, the model is rejected and user should specify a different model. However the user can also refine the already accepted model by adding or removing a certain component. This provides a considerable freedom to control a complexity of the model and gives user a way to find as simple model as possible. The procedure is already implemented in CTSM software. [2] A significant advantage of this method is that importance of adding the parameters can be tested by standard statistical tests, e.g. likelihood-ratio tests [61]. The biggest disadvantage of this method is its computational complexity and inability of handling larger amounts of data.

2.3. *Comparison of the identification approaches*

Finally, Table 2 summarizes the MPC applicability of above mentioned approaches from various viewpoints.

3. Co-simulation based building modeling

In this section we present a methodology how to utilize BEPST to obtain an LTI model for control. The motivation is the following:

- Data collected from the real operation of a building nearly always violate conditions under which the statistical identification techniques estimate models reliably. The main issues are the persistent excitation and the closed-loop nature of the identification data.
- A suitable identification experiment can significantly increase the model quality but such experiments can be quite expensive [11]. In addition, the more system inputs, the longer the necessary experiment time which leads to additional costs.

[2] Continuous-time stochastic modeling [60]

Table 2
Comparison of the identification/modeling approaches.

	Building simulation software modeling	RC modeling – tabular data driven	Deterministic and stochastic semi-physical modeling	Subspace identification	Model predictive control relevant identification
Planning data from architects and engineers need	Yes	Yes	No	No	No
Operation data need	No	No	Yes	Yes	Yes
HVAC engineering background needed	Yes	Yes	No	No	Yes
Result is achieved in defined time	Yes	Yes	No	No	No
Use of prior information about building	Yes	Yes	Yes	No	Yes
Continuous model update	No	No	Yes	No	No
MPC applicable	No	Yes	Yes	Yes	Yes
Estimation procedure computational complexity	–	–	Medium	Low	High

- In real operation, temperature signals suffer from a co-linearity. Physically, this means that the temperatures in the building are very similar in time and make the estimation problem ill-conditioned. Moreover, in case of MIMO systems, this can even lead to wrong input–output coupling in the resulting model.

It is therefore desirable to use BEPST not only for validation of the resulting controller, but for the identification data generator as well. An arbitrary experiment with no financial cost can then be performed in order to achieve a model of a desirable quality. Moreover, the complexity of the model can be controlled, e.g. by means of an examination which sensor is important for the model and which is not. If the BEPST model is a true copy of the real building then the resulting LTI model describes the real building sufficiently precisely.

3.1. Coupling control and building simulation

Even though BEPST are open for custom model adaptation, the flexibility of the tools is still limited – since they were developed and optimized for a building energy performance simulation in the first place. In order to enhance flexibility and combine simulation tools of different emphasis (such as building performance and control), a *co-simulation* becomes more and more important. Co-simulation describes the integration of different tools by run-time coupling. This allows for example to couple building energy performance simulation tools to Matlab, thus provides new possibilities to building simulation. Co-simulation fundamentals for building simulation such as coupling strategies and data transfer are described in [62].

Run-time coupling allows for example simulation assisted control. Different fields of application of building simulation tools concerning building control were defined by [63]:

- Used as an emulator, the simulation tool replaces the building and its plants. BEPST is given input by the simulation. This approach can be used for control product development, tuning control equipment, fault-detection amongst other applications.
- Used as evaluator, the building simulation tool provides a detailed building and plant model for evaluation of different control strategies, evaluation criteria being energy performance and user comfort.
- Coupling the building simulation tools to the BEPST simulation assisted control is feasible. The building simulation tool becomes part of the controller and is used to evaluate control scenarios for each control task before control actions are applied on the actual building.

Yet another field of application for co-simulation is a development and testing the MPC. Currently, many BEPST already feature interfaces to other tools:

- Trnsys allows coupling with Matlab on Windows platforms making use of Type155.
- Extensive capabilities for coupling simulation tools are provided by the Building Controls Virtual Testbed (BCVTB) which is developed by the Lawrence Berkeley National Laboratory [64]. BCVTB is a middle-ware tool that allows to couple different simulation programs for distributed simulation. Programs that can be linked via the BCVTB are EnergyPlus (EP), Matlab/Simulink, Dymola and Radiance. Data exchange with BACnet building automation systems is also featured.

3.2. Combined procedure

In this procedure, we combine benefits of both approaches, i.e. we use BEPST for identification experiments to get input–output data and then we use statistically-based algorithm to identify LTI model from the generated data.

The whole procedure of getting a building model is described in the following steps. Note that in the following discussion, we consider use of EP only, however, an arbitrary simulation tool featuring co-simulation and providing an implicit model can be used.

3.2.1. Choice of model inputs and outputs

The choice of model inputs and outputs plays an important role for the particular identification procedure. They must be chosen so that the resulting underlying physics is linear. The specific selection of the system inputs and outputs is provided in the second case study of Section 4.

3.2.2. Data preparation and system identification

High quality data needed for a system identification (SID) can be obtained as an output of the EP model provided the model is excited by specially designed inputs. The main task of the generator of EP inputs (GenEI, see Fig. 3) is a generation of sufficiently exciting input signals.

Three different kinds of input signals can be considered; pseudo-random binary signal (PRBS), sum of sinusoids (SINE) and multilevel pseudo-random signal (MPRS). Let τ_H, τ_L denote the slowest and the fastest time constants of a system, respectively. Then the frequency

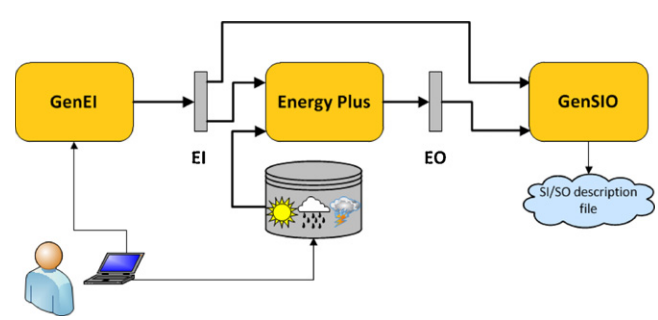

Fig. 3. Preparation of data for identification.

spectrum to cover is (ω_*, ω^*) with $\omega_* = 1/\beta\tau_H \leq \omega \leq \alpha/\tau_L = \omega^*$, where α defines how fast is a closed loop w.r.t. an open loop response. β specifies a low frequency information corresponding to a settling time. Typical values are $\alpha = 2$ and $\beta = 3$, which corresponds to 95% of the settling time [65]. In case of MPRS, an input sequence is computed by Galoise fields [65] with the number of shift registers c and a length q, which defines the maximum possible multiple of harmonics to suppress. Or, in the opposite direction, let h be a maximum possible multiple of harmonics to suppress, then q must be chosen so that $q \geq 2^h - 1$ holds and c is computed from

$$\omega_* \geq \frac{2\pi}{T_s(q^n - 1)}. \tag{32}$$

The length of a signal cycle is $N_{cyc} = q^c - 1$, which (in a time domain) represents a signal of duration $T_{cyc} = N_{cyc}\,.\,T_s$. The number d of signals to be generated does not need to be considered, as it is sufficient to generate a single signal and shift it $(d-1)$ times, which guarantees good statistical properties of the generated signals [66].

Both the generated EP inputs and weather predictions are processed by EP to produce EP outputs. To have a complete set of inputs and outputs for SID, we need EP outputs, some variables from schedules (e.g. building's internal gains, equipment gains) and databases (weather predictions) that are processed by a software block written in Matlab (GenSIO, see Fig. 3).

Finally, having inputs and outputs ready, a SID algorithm is performed to obtain a linear time-invariant model.

4. Case studies

We will discuss here two examples of buildings with completely different structure and complexity. Both of them will demonstrate the properties of the co-simulation based procedure from Section 3. The first one deals with a large office building that is modeled using EP and due to the model complexity, 4SID identification technique is the only option to get LTI model, whilst the second example is an artificial building constructed in Trnsys environment, where the performance of all identification approaches from Section 2 but 4SID is investigated.

For evaluation of a model quality we will use a normalized root mean square error (NRMSE) fitness value defined as:

$$NRMSE_{fit} = \left(1 - \frac{\left\|y_k - \hat{y}_k\right\|_2}{\left\|y_k - E(y_k)\right\|_2}\right) 100\ \%, \tag{33}$$

where E stands for the expected value operator.

4.1. Example I: a large office building in Munich

4.1.1. Building description

The building under investigation is a large office building in Munich (20 000 m^2, six above-ground floors, see Fig. 4(a) and (b)). The objective of the identification is the 3rd floor with an area of approximately 2800 m^2. Based on a usage, a façade orientation and a HVAC supply, the floor can be divided into 24 mutually interconnected zones. The façade of the building has a window-to-wall ratio of approx. 70%. Façades to the atrium have a glazing ratio of approx. 50%. Roughly 50% of the windows have interior blinds, remaining blinds are in-between-glass blinds of double windows.

The building automation system contains several actuators: individually controlled convectors, 24 independently controlled radiant ceiling panels for cooling and heating, two air handling units (AHU) for control of the ventilation, and Venetian blinds for all windows in all zones. Energy supply, i.e. hot and chilled water supply for the entire building, is provided by a central heating and a cooling plant, which is located partly in the basement and partly on the roof. District heating is used for the building's heat supply. Chilled water is provided locally by mechanical chillers. We will now follow the steps from Section 3.

4.1.2. Building modeling

Choiceofmodelingstrategy, inputsandoutputs. Following the discussion in Section 3 we selected the heat fluxes affecting zone temperatures for system inputs and temperatures and illuminances for system outputs to obtain an LTI model. The selection resulted in 288 inputs and 48 outputs lumped into the variable categories as described in Table 3. Note that some inputs are common for multiple zones, while others are unique for respective zones. Signals, that are common for multiple zones are marked by 'No' in the column 'Zone relevant' of Table 3. Signals marked by 'Yes' are unique for each single zone and therefore each category has as many signals as zones (E.g. there are 16 convectors for 24 zones, therefore variable category Q_{CONV} contains 16 signals.). Moreover, all the variables are inputs/outputs/disturbances of the LTI model (model produced by SID). These are not necessarily the same as those of EP (remember a use of GenSIO from Section 3.2.2), which is indicated in the column EP equivalent.

Excitationsignals. The fastest and the slowest time constants are 4 h and 20 days, respectively. The minimum necessary length of the experiment as well as a suitable sampling time and additional settings are obtained from the aforementioned technique (see Section 3.2.2) for the excitation signal generation. Apart from the frequency properties there exist further requirements on the properties of the input signal such as minimum and maximum values, a maximum possible step or a mutual exclusivity of some signals.

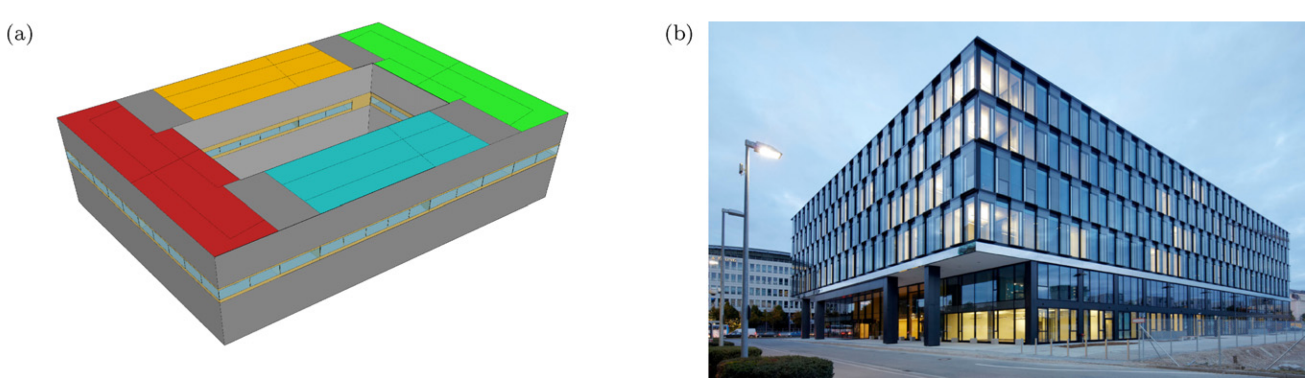

Fig. 4. Office building in Munich. (a) 3D simulation model: investigated zones are on the third floor, other floors are grayed out. The zone layout is shown on top of the model for clarity. The zones of the same sub-system are colored alike. Core areas are gray. (b) A photo of the building shortly before opening.

Table 3
Notation of the variables used for system identification.

ID	Variable category	Type	Zone relevant	EP equivalent
Q_{CONV}	Convector heating rate	Input	Yes	Same quantity, power can be arbitrarily set within limits
ZCPCR	Zone ceiling panel cooling rate	Input	Yes	Supply water temperature and mass flow rate through plumbing can be adjusted. Together with return water temperature, they stand for heat flux of radiant ceiling
ZCPHR	Zone ceiling panel heating rate	Input	Yes	Same as ZCPCR
LG	Lighting gains	Input	Yes	Same quantity, power can be arbitrarily set within limits
NRF	Net radiation flux	Disturbance	Yes	Partly by means of blinds control
FP	Fan power	Input	Yes	Air flow rate (which is either 55 or 0 m^3/h) and supply air temperature. Together with return air temperature, they stand for heat flux of fans.
ODBT	Outdoor dry bulb temperature	Disturbance	No	Same quantity
EG	Equipment gains	Disturbance	Yes	Same quantity
OG	Occupancy gains	Disturbance	Yes	Same quantity
ZT	Zone temperature	Output	Yes	Same quantity
ZI	Zone interior illuminance	Output	Yes	Same quantity

After analysis of a frequency response of the system, PRBS was chosen as the most convenient excitation input signal from the investigated group. Its advantage compared to SINE input signal is that it covers the whole frequency interval (frequency spectrum of SINE signal is not continuous) and compared to MPRS is the speed of its generation (milliseconds in case of PRBS, minutes or even hours depending on the number of inputs and signal length in case of MPRS).

In case of unknown processes and non-linearities in the system, the best choice is to use PRBS for the first shoot and then analyze system frequency response because MPRS has clearly delimited frequency spectrum and some of system modes might not be properly excited.

Analysis of the linearity of EP model. In Section 3.2.2 we have discussed an importance of a linearity of the underlying physics of the process to allow the use of the 4SID algorithm. Hence we need to verify linearity of the data produced by EP, which can be performed according to a definition, i.e.

$$\alpha f(x_1) + \beta f(x_2) = f(\alpha x_1 + \beta x_2). \tag{34}$$

This means, that independent inputs (e.g. convectors in Fig. 5(a) and (b) lower figures, and equipment and lightning gains Fig. 6(a) lower figure) are fed into EP and the sum of corresponding outputs is compared to the response of EP to the sum of the same inputs. The results can be seen in Fig. 5(c) and (d) for convectors, equipment and lightning gains, respectively. The errors between the responses are summarized in Table 4. The growing error in case of a multiple step in input signals can be explained as follows. Linearity tests were intentionally performed at two different outside temperatures, namely 15 °C and 20 °C. The actual zone temperatures are a bit different due to the heat flux (from/to measured zones), i.e. $Q_{ss} - Q_{cool}(T_z) = Q_{EP}$. Q_{ss} denotes here the heat flux corresponding to the designed input (e.g. convectors), $Q_{cool}(T_z)$ is a flux altering (an actual size depends on the temperature difference between outside and zone temperatures) the requested value and Q_{EP} is a real value of the flux affecting EP. When summing-up two signals of a different step size, there is a different alternation by $Q_{cool}(T_z)$, hence a small difference between the sum of responses and a response of the sums. Nevertheless, it can be concluded, that EP response on selected inputs is indeed linear.

Table 4
Temperature linearity error.

Errors in %	EG	LG	Q_{CONV}	
			15 °C outside	20 °C outside
2nd step	3.9	2.1	4.4	5.2
3rd step	3.3	1.0	3.4	5.0

Settings of the identification procedure. Final step is the choice of parameters for the SID, namely identification algorithm, desired model order and size of the Hankel matrices.

1. **Identification algorithm:** There are several algorithms covered by 4SID, which differ in, for instance, applicability, numerical stability and computational demands [21]. For our case, N4SID was selected.
2. **Desired model order:** Although the order selection has already been implemented in N4SID, an insight into a building physics can help. A physically based order selection leads to a 2nd–3rd order dynamics per output temperature [57] leading thus to the order between 48 and 72 for 24 zones. After employing N4SID algorithm and validation tests, 72th order model (order selection according to an algorithm Eq. (8)) turned out to be indeed a good choice, considering both its simplicity and sufficient precision.
3. **Size of Hankel matrices** is given by the number i of block rows, Section 2.2.1, $i > n$, where n is a system order to identify [21]. Essentially, i means how far into the past/future of the measured data is searched. It may therefore seems that bigger i leads to a better result. However, one should not forget, that the size of the system matrices grows considerably with the system size and i must be therefore a trade-off between computation difficulties and the size of a "memory window".

   Several values of i were examined experimentally and the results for step responses to several inputs for i = 24, 30, 36 and 40 are depicted in Fig. 7. All step responses recorded satisfactory results as far as reliable dynamics, a sign of the effect and nominal value is of concern. The increase of i leads only to DC-gains change. Next, the measured step responses were analyzed. It turned out, that with bigger i the model step responses approach to the measured step responses (see Fig. 8). Because of the computational limitations, the i = 40 has been selected as a suitable choice for the size of Hankel matrices.

Prediction properties. The good prediction properties of the identified model are crucial for predictive controller. For comparison of the predictions for various prediction horizon refer to Figs. 9 and 10. It can be seen, that the model has satisfactory prediction properties even for larger horizons.

4.2. Example II: artificial building modeled in Trnsys

In the second example, we will consider a small building modeled in Trnsys environment. As mentioned before, the

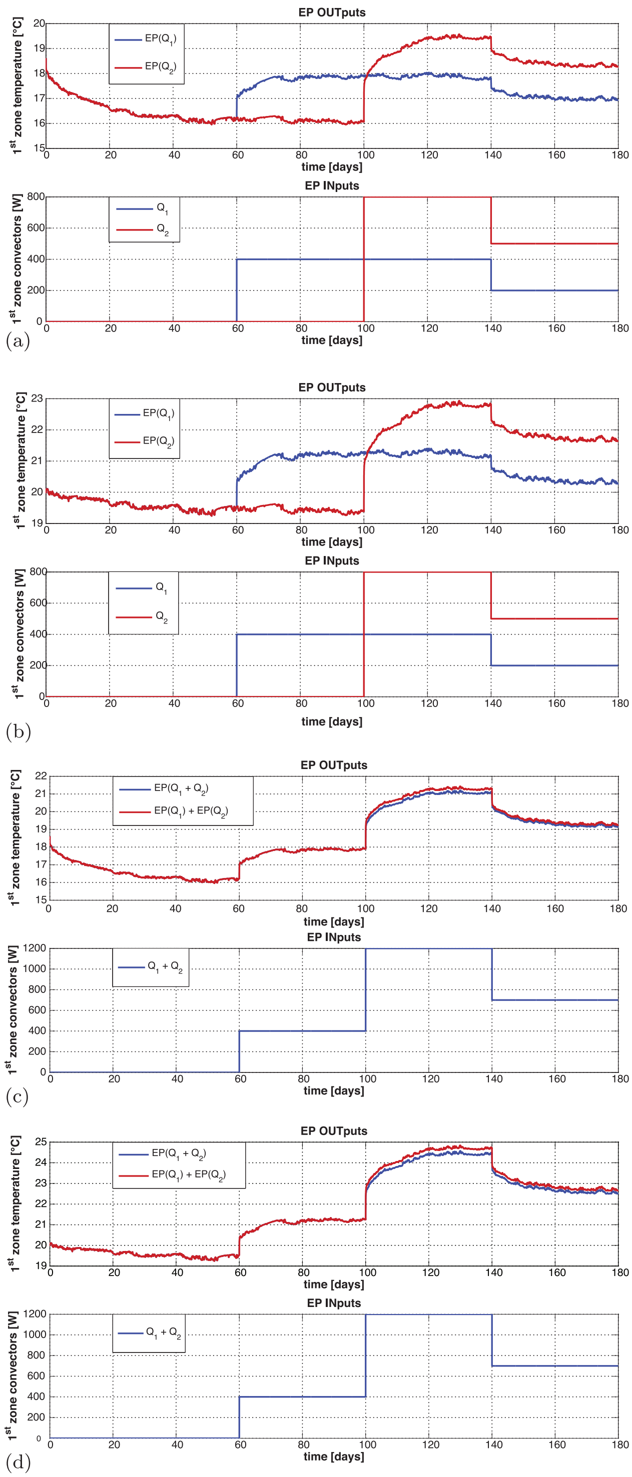

Fig. 5. Convectors: test of linearity of the EP model. (a) Two convectors: input signals at outside temperature 15 °C and the EP model response. (b) Two convectors: input signals at outside temperature 20 °C and the EP model response. (c) Sum of two convectors: input signal at outside temperature 15 °C and EP model response (response of sum and sum of responses). (d) Sum of two convectors: input signal at outside temperature 20 °C and EP model response (response of sum and sum of responses).

current system identification techniques from Section 2 can be used.

4.2.1. Building description

A building, schematically outlined in Fig. 11, was constructed in Trnsys environment. It is a medium weight office building with

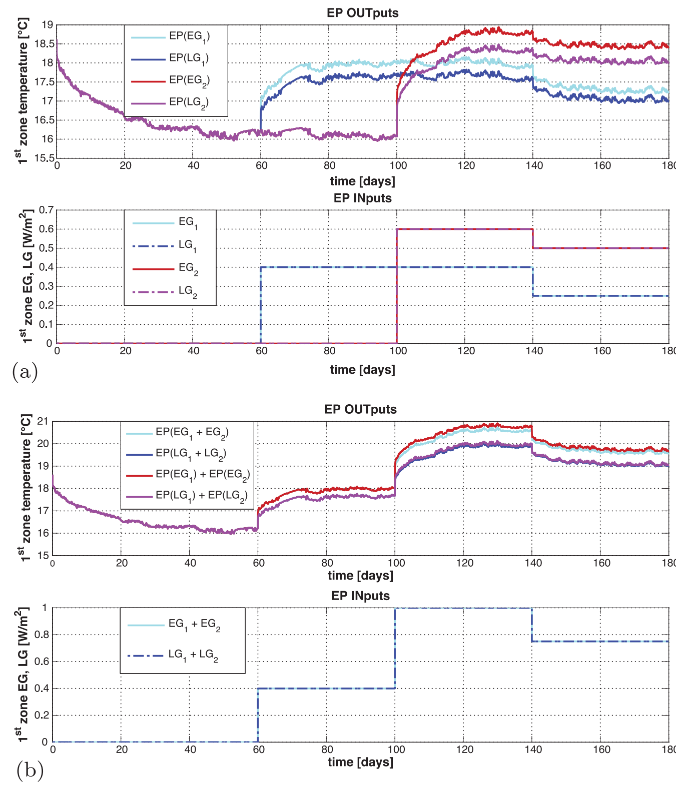

Fig. 6. Equipment and lightning gains: test of the EP model linearity. (a) Two convectors: input signals at outside temperature 15 °C and the EP model response. (b) Sum of signals: input signals at outside temperature 20 °C.

two zones (5 m × 5m × 3 m) separated by a concrete wall (involving the transient properties between zones). South oriented walls of the zones include a window (3.75 m^2). The HVAC system used in the building is TABS [67]. Technically, a set of pipes is placed in the ceiling and distributes supply water which then performs a thermal exchange with a concrete core. Each zone has a unique heating circuit with a constant mass flow rate of the supply water leaving thus a supply water temperature the only manipulated variable. This control strategy was chosen to mimic a real-life application [12], where there are no valves, thus no possibility of control the fluxes.

We employed several Trnsys components such as (i) Type56 for a construction of the building, (ii) Type15 for outside environmental conditions (involving ambient temperature, outside air relative humidity and solar characteristic) with year weather profile corresponding to Prague, Czech Republic, (iii) Type155 to establish a link between Trnsys and Matlab.

The communication link was used to generate identification data in order to excite the system properly. Based on the previous discussion, PRBS was used as an excitation input signal. Time-step of the simulation was set to T_s = 0.25 h which guarantees a proper convergence of Trnsys internal algorithms and is also suitable for a description of important building dynamics.

4.2.2. Building modeling

Choiceofmodelingstrategy, inputsandoutputs. The model built in Trnsys has 18 states and 12 inputs (manipulated variables and disturbances). We applied an iterative procedure for selection of a minimum input and state sets [42]. The procedure iteratively selects only those inputs and states which brings statistically significant information to the model. Finally, we obtained 4 out of original 12 inputs (8 disturbances were proven not be significant),

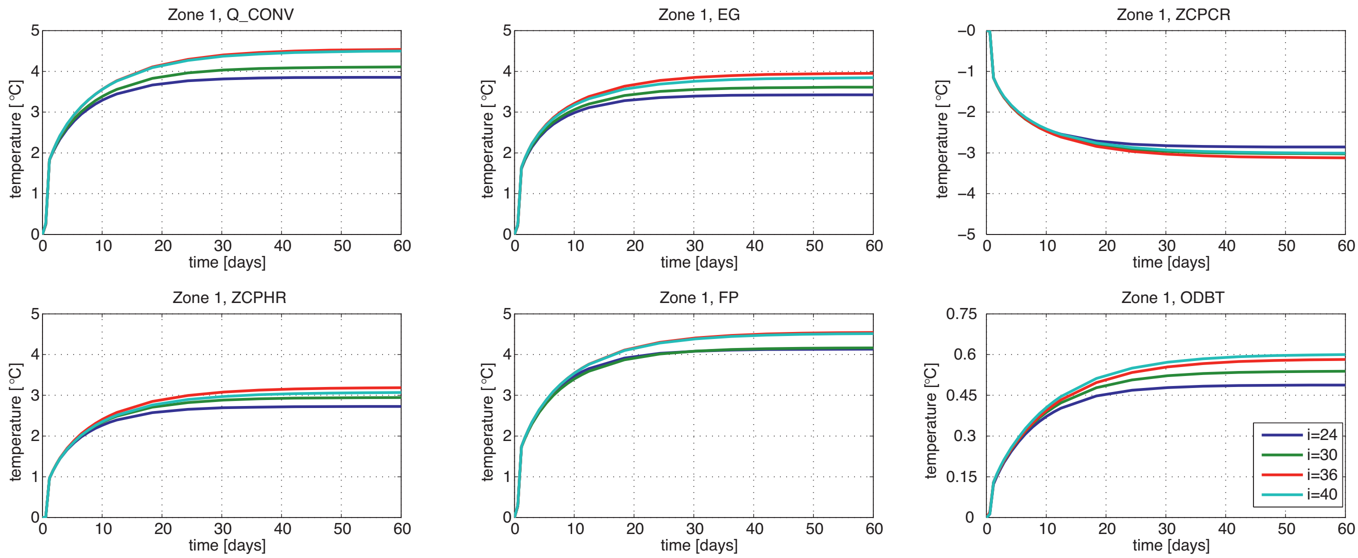

Fig. 7. Step responses of several inputs in zone 1 for different *i*s. Vertical axes are particular contributions to zone temperatures.

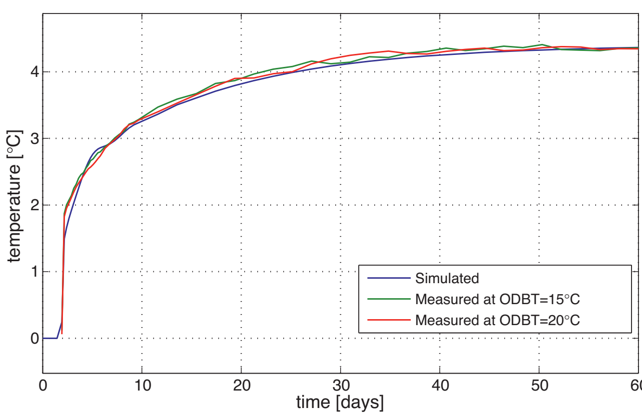

Fig. 8. Comparison of measured and simulates step responses from convectors.

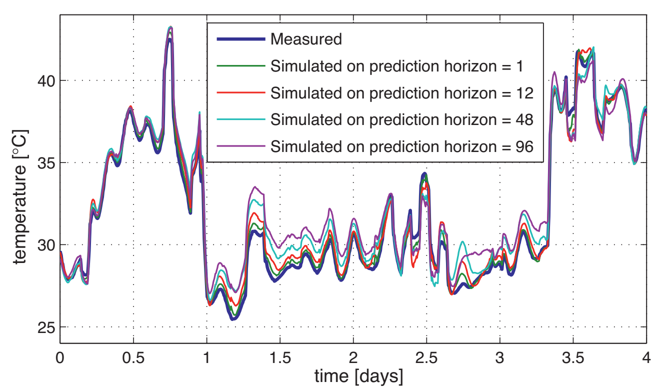

Fig. 10. Part of model outputs.

namely T_{sw_1}, T_{sw_2}, T_o and $\dot{Q}$ and 6 out of original 18 states, namely T_{c_1}, T_{wall_1}, T_{z_1}, T_{c_2}, T_{wall_2} and T_{z_2} with a meaning described in Table 5. Note that the table presents only those states that are depicted in Fig. 11.

Identificationproceduresused. We are now ready to investigate the applicability of the methods from Section 2. As the investigated building is a small, more computationally demanding identification techniques can be used.

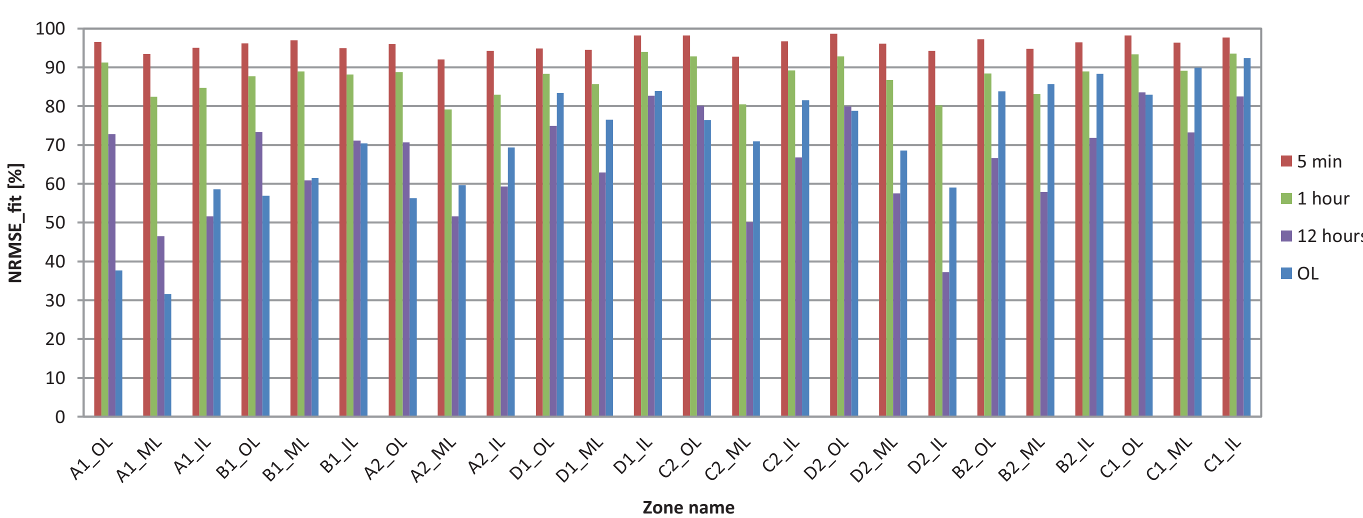

Fig. 9. $NRMSE_{fit}$ for all zones for different *k*-step ahead predictions.

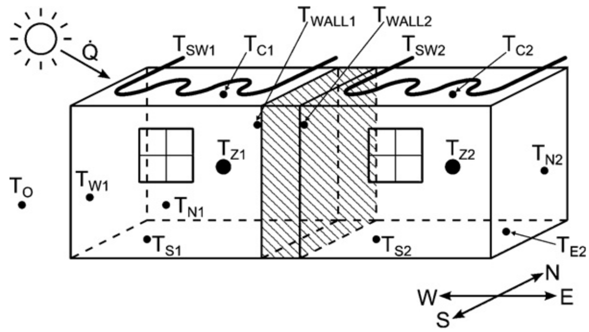

Fig. 11. A scheme of the modeled building.

Table 5
System states, inputs and measured disturbances.

Notation	Description
(a) System inputs and measured disturbances	
T_{sw_1}	Supply water temperature, zone 1
T_{sw_2}	Supply water temperature, zone 2
T_o	Ambient temperature
$\dot{Q}$	Solar radiation
(b) System states	
T_{c_1}	Ceiling core temperature, zone 1
T_{wall_1}	Core temperature of common wall, zone 1
T_{s_1}	Core temperature on south side, inside, zone 1
T_{w_1}	Core temperature on west side, inside, zone 1
T_{n_1}	Core temperature on north side, inside, zone 1
T_{z_1}	Zone temperature, zone 1
T_{c_2}	Ceiling core temperature, zone 2
T_{wall_2}	Core temperature of common wall, zone 2
T_{s_2}	Core temperature on south side, inside, zone 2
T_{e_2}	Core temperature on east side, inside, zone 2
T_{n_2}	Core temperature on north side, inside, zone 2
T_{z_2}	Zone temperature, zone 2

For MRI and DSPM, we consider a model of the following form

$$y(z) = G(z)u(z) + H(z)e(z), \tag{35}$$

with $G(z)$ and $H(z)$ transfer functions corresponding to a deterministic and a stochastic[3] part of the system. A state-space model will be use for case of PSPM as:

$$dx_t = (A(\theta)x_t + B(\theta)u_t)dt + \sigma(\theta)d\omega_t, \tag{36}$$

$$y_t = C(\theta)x_t + D(\theta)u_t + e_t, \tag{37}$$

where $\theta \in \Theta \subset \mathbb{R}^p$ is the vector of parameters, ω_t is the n-dimensional Wiener process and $e_t \sim \mathcal{N}(0, S(\theta))$ is a white zero-mean Gaussian noise and $A(\bullet)$, $B(\bullet)$, $\sigma(\bullet)$, $C(\bullet)$, $D(\bullet)$ and $S(\bullet)$ are appropriate system parametric matrices.

***Predictionproperties*.** The performance of the respective methods is evaluated using $NRMSE_{fit}$ and is summarized in Fig. 12. To show the properties of the identification methods, Fig. 12 depicts results which correspond only to the deterministic transfer function. A comparison of measured and predicted outputs (for deterministic transfer function only) obtained from models of different approaches for 15 steps-ahead prediction is depicted in Fig. 13. These results are presented for one zone, however, they are almost identical to the other zone which is not presented due to space reasons. Note that 4 step-ahead predictions for all methods recorded $NRMSE_{fit}$ over 96% even without stochastic part which can be considered as excellent. The growing error with larger horizons

[3] Note that $H(z)$ includes non-linearities of a detailed Trnsys model which cannot be described by a linearized model.

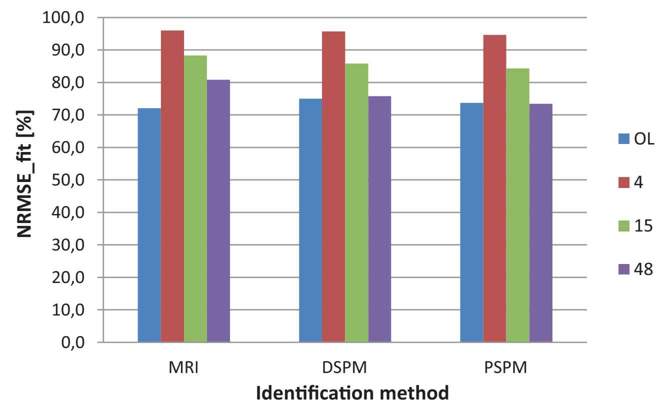

Fig. 12. $NMRSE_{fit}$ for different methods (MRI, DSPM, PSPM) and different prediction horizons (4, 15, 48 steps and open loop, each step is 15 min).

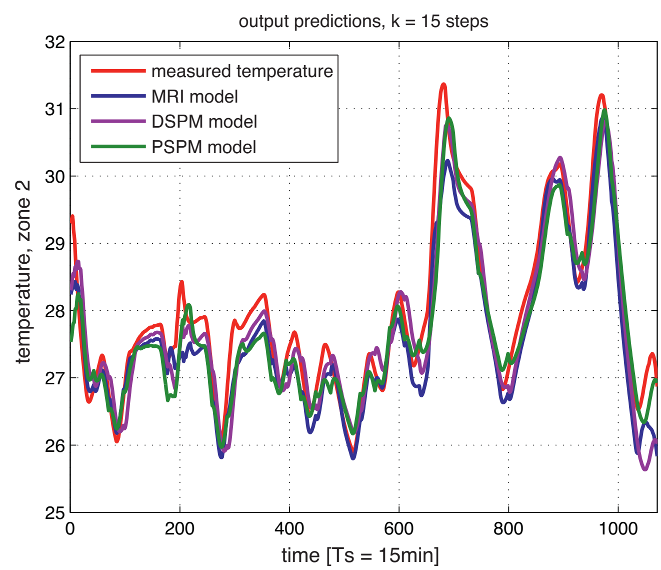

Fig. 13. Temperature predictions using different methods and 15 step-ahead prediction horizon.

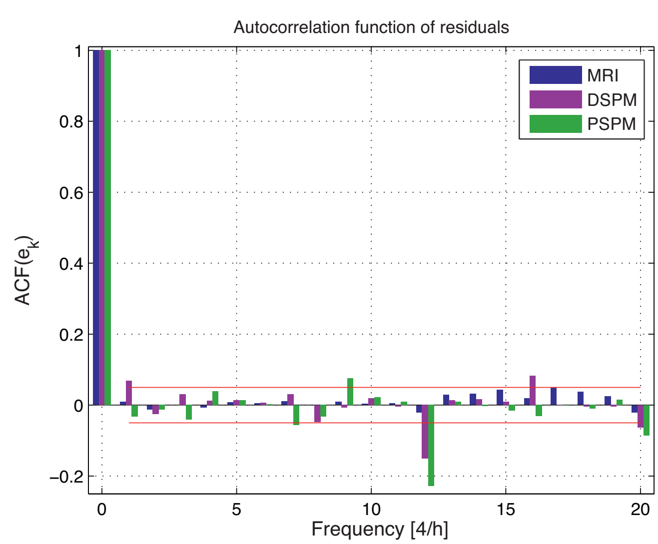

Fig. 14. Autocorrelation function of residuals of predicted output. The red horizontal lines correspond to 5% significance level. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

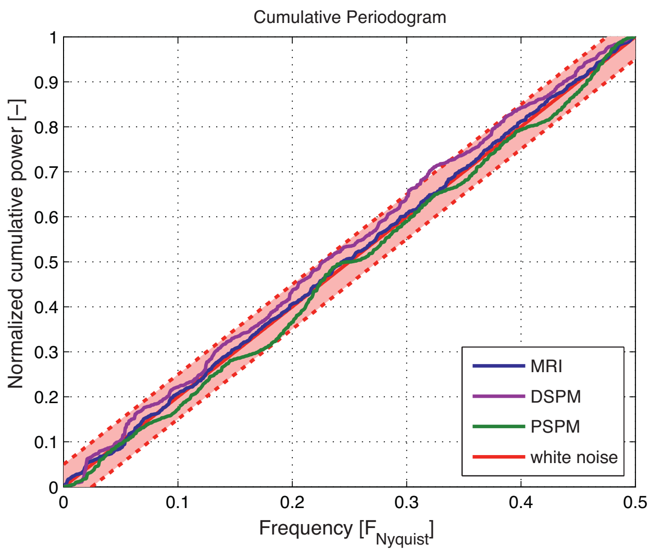

Fig. 15. Cumulative periodogram of residuals of a predicted output. The red dotted lines correspond to 5% significance level. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

is caused by non-linearities of the underlying physics, which are lumped in a stochastic part ($H(z)$).

Validationofmodels. To evaluate the validity of the models we have used several tests applied to model residuals, namely (i) test of AutoRegressive (AR) process order, for details refer to [68,69] and (ii) tests using partial autocorrelation function (PACF) and cumulative periodogram [70].

All the tests confirmed the whiteness of the residuals for all three identification approaches, for visual results refer to Figs. 14 and 15. It can be seen, that the residuals are well within the confidence intervals corresponding to the 5% significance level.

5. Concluding remarks

Apart from a detailed overview of modeling approaches and algorithms suitable for a predictive control, two case studies were presented. The first was a real-life example of a large office building in Munich where a new procedure combining an implicit model built in EnergyPlus and a subsequent statistical identification, namely 4SID algorithm, was presented. For large buildings with a complex structure, the only viable option seems to be statistically-based algorithms which are inherently capable of treating MIMO systems. The biggest disadvantage of 4SID is that it does not preserve a physical structure during modeling phase, which causes deteriorating predictions for the larger horizon.

The second example of an artificial building modeled in Trnsys demonstrated that use of a number of identification approaches (MRI, DSPM, PSPM) led to the very similar results. These methods make use of a known system structure and estimate system parameters. This property ensures appropriate prediction properties even for longer prediction horizon. On the other hand, time demands and a computational complexity become the issue for these methods in two ways. (i) With a growing complexity of a process (building), a description becomes difficult to follow pointing thus to 4SID as the only suitable candidate. (ii) Use of a probabilistic semi-physical modeling and MRIs for large datasets and/or complex systems becomes computationally infeasible.

Therefore, the methods that make use of a physical description of a system should be used primarily for buildings with simpler structure.

Note that not all the BEPST as introduced in Section 2.1 can be used in co-simulation as not all of them posses the capability of the co-simulation. Building modeling tools Trnsys and EnergyPlus were used to mimic the behavior of a modelled building. All the presented models are in explicit form with reasonable prediction properties suitable for predictive control.

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